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The fractal nature of the everyday environment

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Abstract. If the size of a space is measured by counting the number of places available for a particular activity, rather than by using an absolute measure such as the square metre, then small people may find the world larger. Size measured by number of places becomes a function of the size of the user, and the form of this function suggests that the everyday environment has a fractal dimension, a single number which may be associated with architectural qualities. This measure was tested with an experiment based on children playing hide and seek and was used to explain some illusions of perception of size.

Introduction

Why do scenes of our childhood so often seem strangely smaller than we remember them? The obvious explanation—that we experience our growth as the world shrinking—may not be a complete answer. In my experience, the illusion is more pronounced in some places than others which, if correct, indicates that the ability to appear larger to a child than to an adult may be in some way embodied in the environment. For example, schools and churches seem to be particularly affected whereas roads and open fields appear less changed, and although some larger objects like furniture seem to shrink many small objects like toys and books do not. Interestingly, the print in books not seen since my childhood does not seem smaller now than when I first read them. What could be the cause of these differences?

Could it be that we are correcting an immature assessment of size we made as children? Many things are sized differently for children and adults: children's bodies are proportioned differently, and although toys may be smaller than the things they represent other objects are enlarged and rounded to suit the infant's grip. In parallel with these differences children's perception of time also seems to be different to an adult's; most people agree it flows more swiftly as we get older. But, appealing though this explanation may be, the sensation of shrinking can still be felt with spaces known only in teenage years which could count against the illusion being the result of an undeveloped understanding. Nearly everybody revisiting their old secondary school feels it to be diminished even though they would have been near enough adult size when they left.

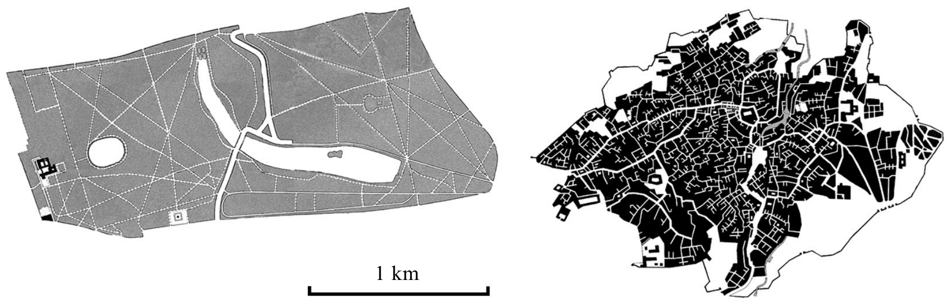
We interpret depth in spaces by seeing which objects obscure those further behind, with clues from perspective and binocular vision. That these factors complete our methods of reading a space is known because it is possible to fool a viewer by using an artful arrangement, such as a trompe-l'oeil, which provides ambiguous readings of these factors (Gregory, 1998, pages 60 – 66). From these sources of information we form a judgment of the layout and size of a space, but it is doubtful that we do this by constructing a mental scale-plan which could be compared with the map of somewhere else. After all, it is notoriously difficult to draw a plan of a space without surveying it, even one we know well. Rooms may be skew and still appear rectangular, we may be deceived by size constancy effects, and it is very difficult to guess distances accurately.

So it is very likely that we do not read spaces in terms of an absolute measure, yet the ability to assess the size of a space is not a skill which anyone feels they lack. How then do we do it and why then do we consistently overestimate the size of spaces of our childhood?

An activity-based method of measuring space

The theory proposed here is that the shrinking is no illusion at all, but that the world really does get smaller as we grow. Our innate measure of space is a function of our size, because it is in part based on *counting*, that is, on estimating the number of places in a space which may be occupied, either in reality or in our imagination. Our ability to estimate size is therefore related to our ability to estimate numbers. If this is the case then the number of places will be a function of our own size because children will happily occupy places that adults will not consider on account of their size and dignity. Differences in perceived size between children and adults will therefore be most pronounced in things which are used in different ways by the two groups. Books and toys are handled in basically the same way by both groups, but an armchair in which an older person will simply sit is a field of opportunity to a child who will live underneath it, sit on the arms, use the back as a wall against which to play, and so forth. That is why the old chair seemed larger all those years ago.

What follows is an attempt to quantify the impression that the size of a space is related to the number of opportunities it affords for a particular activity and produce a measure of size more in accordance with our intuition. This may be illustrated by comparing Hyde Park in London with the old city at Fez, places which are roughly equal in area (as shown in figure 1) but seem very different in size. Although the width of Hyde Park may be encompassed at a glance, Fez is a prodigious maze that seems to go on forever in which, without a guide, you will quickly be lost. This difference in complexity may be expressed as a difference between their fractal dimensions, which is measured by counting the number of cells which are built up and the number which are open. London, for example, gives a value for its dimension of about 1.7, a number which increases over time as more of the city is filled in (Batty and Longley, 1994). On this reckoning, the built-up portion of Fez, whose plan here shows only the larger passageways, would be a highly developed city with a high fractal dimension and Hyde Park would have a very low value as it is mostly open space. The complexity of a space is related to its ability to accommodate many different activities. Of course, it is true that if we count places to stand in a square metre of ground then Hyde Park would hold a larger crowd than built-up Fez. But for most activities—such as 'places to sit and eat on a damp day'—Hyde Park will provide only as many places as there are benches to sit on whereas Fez would be much more accommodating. Similarly, if we count



(a) (b)

Figure 1. (a) Hyde Park and (b) the old city at Fez to the same scale.

'places to wait to meet someone', and restrict ourselves to nameable places, corners, and so forth, Fez would offer immensely more possibilities since it would be peculiar to stand in the middle of an open grassed area. Fez may be crossed in many different ways without ever repeating your route and, although there are many routes across Hyde Park, such as a man methodically mowing the grass might follow, to a pedestrian they are hardly worth distinguishing.

Testing the theory is difficult if we count places which may be occupied only in our imagination as well as those where we may actually live. The eye, so to speak, admits no dimension and in one's imagination one may occupy inaccessible places such as on the ceiling, in wallpaper, on top of furniture, and all these places will contribute to the feeling of spaciousness for a person confined to a chair as much as they do for a child crawling about the floor. To test the theory in a repeatable way it will be necessary to define 'place' more precisely: for example, as a place to read, or a place to hide. To what extent counting these places will give an indication of the number of all the places that contribute to the size of a space is a matter of opinion. One shortcoming is that these restricted definitions mostly count places in plan and so do not necessarily give a measure of inaccessible places occupied in the mind above ground level. Notwithstanding this drawback of using a simple definition of place, it will be shown that the number of places is a function of the size of the user, a result that is interesting in its own right.

Fractals and complexity in the built environment

Mandelbrot (1983) created fractals by repeating and multiplying a motif at different sizes according to simple algorithms. Some of them, notably the Menger Sponge (illustrated in figure 2), have architectural qualities; indeed, at least one building proposal has been based on the sponge (Bolles-Wilson, 1993). Objects can, however, be fractal without being generated by a rule—it is enough that they repeat some motif over several scales of size, a quality possessed by both classical and gothic buildings, and by the Eiffel Tower (Mandelbrot, 1983, page 131). In fact, it is sufficient to show that something as loosely defined as 'an element which will fit in a box of particular size' repeats and multiplies at smaller and smaller scales. From this point of view almost any building can show fractal qualities, one simply has to count the elements

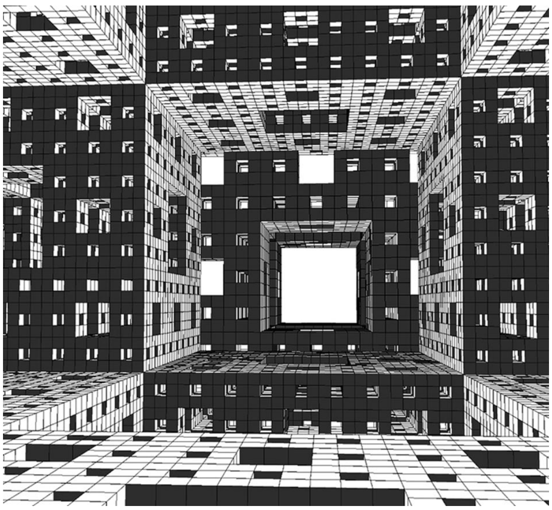


Figure 2. The Menger Sponge.

of a facade which occur within different ranges of size and see how they increase in number as they get smaller. Buildings with a lot of detail will have a higher fractal dimension than those which are plain. The pioneer in this work was Kiemle (1967) who applied information theory to architecture with a view to finding a measure of the complexity of a building. His motive was to justify the preservation of older buildings, even those with no special historical or artistic merit, on the grounds that they were more complicated and interesting. For this purpose, he developed a scientifically based definition of complexity that involved counting nameable elements (windows, pilasters, brackets, and so forth) falling within certain size limits. His method was applied in a study of Munich by von Buttlar et al (1972, pages 65 – 71). Unfortunately it was not possible at that time to express the results in terms of fractals. Similar studies which have taken this step have now been done, notably by Bovill (1996) who, in the notation of this article, analysed scaling effects in drawings of buildings by counting N elements in a particular grid size r, and then plotted $\log_{10} N$ against $\log_{10} 1/r$, the gradient of the resulting line being interpreted as a dimension. The relative multiplicity of architectural elements in a particular size range has recently been described as being subject to a universal law (Salingaros and West, 1999) which, it is claimed, is obeyed by the majority of architectural and urban styles, with the exception of those of the 20th century. The law, again adapted to the notation of this article, is that $\log_{10} N \propto \log(1/r)$, with a constant of proportionality (in other words, the fractal dimension) between one and two. The results described below tend to confirm the findings that inverse power-law distributions such as this are ubiquitous in the built environment. These studies dealt with objects considered as being made up of elements of varying size; here the approach is extended to space itself.

An alternative definition of size and dimension of a space

The size of a space will be measured by a pure number, N, the number of places where a particular activity may take place for a person or thing of size r. The size, r, will be some characteristic length in metres, for a person this will usually be their height. Since N depends on the activity and on r it may be written as a function: $N(\text{activity}, r)$.

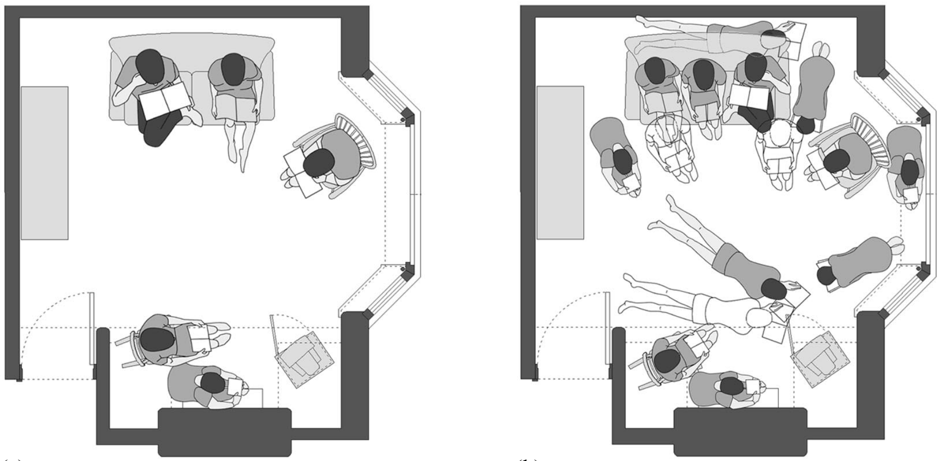
The number N bears some relation to our intuitive understanding of size; given two spaces with the same area in square metres, the one with the larger N for some activity will seem more spacious. A graph of N against r will usually show N increasing rapidly as r decreases, the rate being related to the complexity of the space. This is made clearer by plotting $\log_{10} N$ against $\log_{10}(1/r)$. The gradient, D, of the resulting line may then be interpreted as a dimension of this abstract activity space. The notation and method of calculating D used here are adapted from Mandelbrot (1983, chapter 2).

The method may be illustrated with a trivial example. For an empty room calculate the number, N, of close fitting carpet tiles, whose side is r metres. An experiment is hardly necessary since Nr^2 is the area of room in square metres, and therefore $\log_{10} N = 2 \log_{10}(1/r) + \text{constant}$. $D = 2$, and so the room is two-dimensional.

It is reassuring to see that in certain circumstances D coincides with the traditional Euclidian definition of dimension. This will certainly happen where we are dealing with close packing of similar objects, but in general D will be fractional, that is, a fractal dimension.

Children and adults, different ways of reading a book

How many places may be found to read a book in a room for people of different sizes? To answer this question three children aged 8, 10, and 12 years old, about six of their friends, and two adults were unknowingly observed over two months, reading in a living room of size $4\,\text{m} \times 36\,\text{m}$, furnished with two chairs and a sofa which were not



(a) (b)

Figure 3. Positions chosen by (a) adults and (b) children to read in an ordinary room.

moved substantially during this period. The subjects were observed discreetly on an ad hoc basis so as not to make them self-conscious about where they came to rest. They proved to be fairly conservative in choosing where to settle, generally orientating themselves in relation to the fire or television. Visitors rarely added to the repertoire of places, generally following the lead of the residents. After a while no new positions were observed, the places they used are sketched in figure 3 (lying on the sofa is omitted for clarity). Note that, in this study, N is not the number of people who could perform some activity simultaneously but the number of ways one person could do it; that is, some of the places may overlap. How do we decide if similar places should be counted as two? Since people will generally sit in relation to other objects (a fireplace, a window, and so on) we ask whether such a relationship has fundamentally changed. For example, lying on the sofa facing the window is counted separately from facing the other way. Two windows in a room instead of one may, in this reckoning, double the number of places. The results were as follows:

children, average height $r = 1.2\,\text{m}$, $N = 18$ places;

adults, average height $r = 1.7\,\text{m}$, $N = 6$ places.

The children found three times as many places as the grown-ups which shows why even quite large children might find a room larger than an adult. A calculation gives:

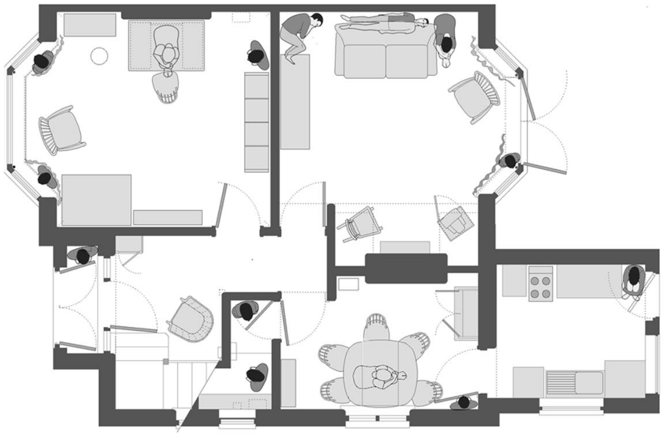
$$D = \frac{(\log_{10} 18 - \log_{10} 6)}{\log_{10}(1/1.2) - \log_{10}(1/1.7)} = 3.15\,.$$

This is a high value; if the dimension had been 2 then the six adult places would have become twelve places for children. Here an objection may be raised—namely, that the children found more space not only because they were smaller but also because they were happy to read lying on the floor. So, although the observations may show why the room would seem larger to children, it would be inappropriate to deduce anything about the how the space varies with the size of the user because the two groups were behaving in different ways. To avoid this difficulty it is necessary to choose an activity which allows the definition of place to be clear, and for which children and grown-ups behave in a similar way. From such an experiment it may be possible to learn something about the nature of the space since we may calculate its dimension, a number which is a measure of its complexity. All this can be done by playing hide and seek.

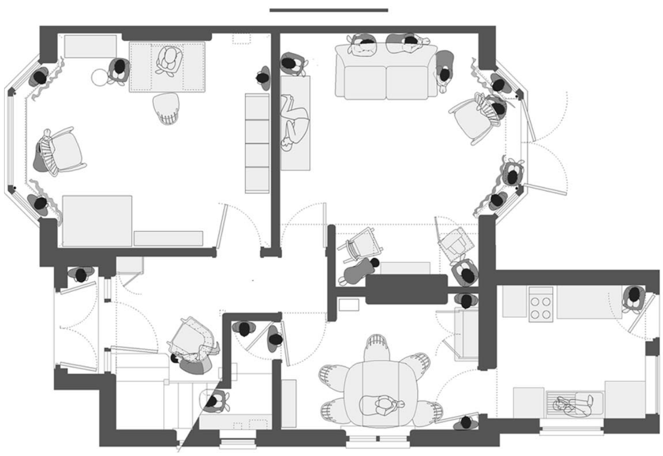
An experiment to measure the dimension of an ordinary house

The experiment was played as a game of hide and seek: six children and two adults were invited to hide and the number of different places they found were counted and recorded against their height. The players learned from each other what places were possible and were playing a game rather than performing an experiment. The game exhausted itself when no fresh places could be found and the game degenerated into the seeker guessing which of a set of known hiding places had been chosen. Figure 4 shows all the places found by children and adults on the ground floor of an ordinary house. The results in table 1 show N (the number of places to hide) and the player's height, r. Not surprisingly, smaller children did better. Figure 5 shows $\log_{10} N$ plotted against $\log_{10} (1/r)$; the gradient, D, is about 2.4.

How accurate are the figures? The bars on the graph in figure 5 represent N plus or minus one space. If the curtains in the right hand room had not gone to the floor the grown ups would have lost a place, since they could not squeeze on to the windowsill like children. If the adults are deducted one place the line may be drawn more steeply, changing its gradient to perhaps 2.6, which gives an indication of how, in this test, fairly insignificant variations in furnishings may affect the dimension. With this small



(a)



(b)

Figure 4. The places chosen by (a) adults and (b) children to hide on the ground floor of a house.

Table 1. 'Hide and Seek': the number of hiding places, N, found for people of height r metres.

	Height, r (metres)	Number of hiding places, N	$\log_{10}(1/r)$	$\log_{10}(N)$
Adult	1.8	10	−0.26	1.00
Child 1	1.6	14	−0.20	1.15
Child 2	1.5	16	−0.18	1.20
Child 3	1.3	23	−0.11	1.36
Child 4	1.2	27	−0.08	1.43

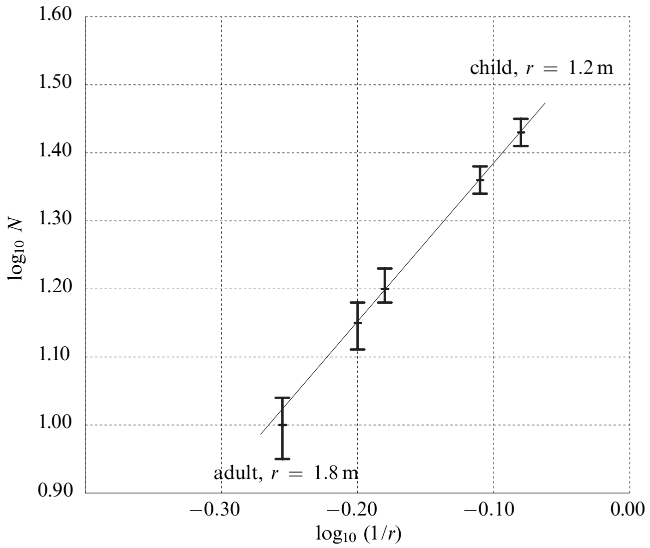


Figure 5. Graph of $\log_{10} N$ against $\log_{10}(1/r)$.

sample, the best that can be said is that the dimension for hide and seek was 2.4 ± 0.2, it will be safer simply to say that its value is certainly greater than 2, which is to say that as you shrink, everyday space multiplies itself inwardly more rapidly than you might expect. This gives a formula $\log_{10} N = -2.4 \log_{10} r + 1.6$ for hide and seek on the ground floor of this particular house.

Hiding successfully depended on hiding around furniture and the game would make little sense in an empty house, even less in an open field, which is to say that filling some of the space with furniture creates places, in sense, creates space. This may serve to explain the illusion often experienced by architects visiting a part-finished building, namely that rooms sometimes seem smaller than expected. Clients must sometimes be reassured, quite correctly, that it will seem larger when decorated and furnished. It will also explain why rooms seem larger when they are warm and when wall-to-wall carpeting is fitted because there are more comfortable places to live, and why demolition sites often seem smaller than the buildings which stood upon them.

The effect on space of getting smaller and larger

An attempt to continue the experiment by counting the number of places where a 50 mm high toy figure could be concealed in one of the rooms was unsuccessful because the number of hiding places was so large that it did not seem that they could every be enumerated, even when the toy had to be in plain view. The ability of the everyday environment to house small objects would be better tackled by counting how many objects there are in a house within particular size ranges, and this will be the subject of a separate study. The numbers involved are large; for example, the left hand room did not appear crowded, yet it had 950 books on open view and the cupboard in it contained about 400 small objects and ornaments. The hide and seek formula would predict 50 000 hiding places for the toy on the ground floor of the house which seems high, but perhaps not unbelievable since, taken together, those places would occupy about one cubic metre. Whether the formula continues to hold true beyond the range of size of human beings is an open question, but it cannot be doubted that the everyday environment can house enormous numbers of small articles. It may be reasonable to suppose that the ability to house personal articles will contribute to a feeling of spaciousness in a room, which will be significant for children's appreciation of size when they turn odd corners into imaginary houses and roads for dolls and toy cars. Small spaces are rarely given any special architectural expression, the notable exception being gothic buildings which can reproduce pointed arches at scales covering up to three orders of magnitude of size—from 30 000 mm to 30 mm—using perhaps seven different orders of size. In this they provide a unified architectural expression of the fractal nature of space, from the size of personal articles to the size of spaces to house hundreds of people.

The process by which space increases inwardly as you get smaller is just as interesting in reverse. Continued out of doors, the size of people may be effectively increased by putting them in cars. The part of the house that provided 15 hiding places for an adult 1.8 m tall had an area of 60 m^2; the area required to provide 15 hiding places for a car 4.2 m long is vastly larger. To play hide and seek in cars, as police and criminals sometimes do, requires a small town to make the game work. If the car is confined to public roads, that is, not on private drives, hiding as successfully as the child in a cupboard is impossible, but round corners in dead ends might be judged to correspond to hiding behind furniture. It is difficult to test this by direct

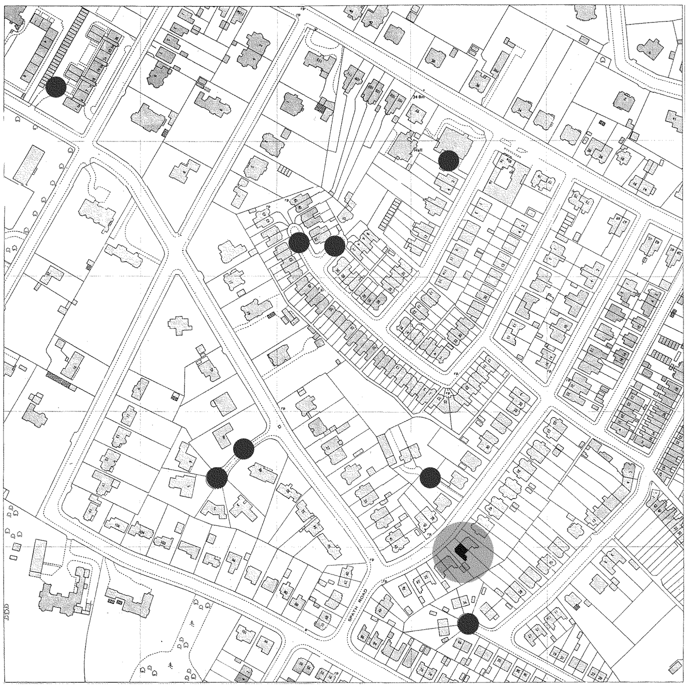


Figure 6. Eight dead ends providing hiding places for about twenty cars on public roads in a 500 metre square.

experiment, but an estimate may be made with a map (figure 6), in this case based on a 500 m × 500 m block around the hide and seek house. In that area, which contained 406 houses, perhaps 20 such places may be identified. Even if hiding in private drives is allowed, this would be far short of the 5000 places which would be expected if the hide and seek formula held true outside the house. This serves as a crude illustration that the fractal dimension of the everyday environment for motorists must be very much less than for people on foot. It is a modern platitude to say that the world is shrinking, for people in cars it may indeed be true.

Evidence that counting places provides a valid measure of space

In a study of the relation between population density and body size in mammals, Damuth (1981) throws some light on the dimension of the natural environment. He examined herbivores between 1 g and 1000 kg in weight and found that, if P = population density (number per km^2), and W = body mass in grams, then $\log_{10} P \propto \log_{10} W$.

Values of the gradient were found to lie between -0.56 and -0.95, and varied according to the type of habitat such as desert, grassland, or forest; high values indicating high population densities for a given size of animal. The average gradient was 0.75, and so he gave the formula:

$$\log_{10} P = -0.75 \log_{10} W + 4.23.$$

Although he did not explicitly express the slope of his graph as a dimension, his results may be compared with those here if we suppose that the weight of an animal is proportional to some characteristic length, r, cubed. The number of creatures living in a given area is then proportional to $(r^{0.75})^3 = r^{2.25}$, giving a dimension of $D = 2.25$—a result which has been extended from bacteria to large mammals (McMahon and Binner, 1983, page 228; Peters, 1983, pages 165–169). Damuth was interested in the energy used by a species and used his result to demonstrate that the energy consumed by all the animals in an area is a constant independent of their body size, but can his formula be given a geometrical interpretation? If size is a matter of counting the number of places where a creature may live, the definition used here, then Damuth's results do indicate that space expands inwardly with a dimension of 2.25 to accommodate smaller creatures. McMahon interprets the result to mean that the larger the organism the greater the mean distance between individuals. Measured in metres this is certainly true, but if we measure the distance in units of the creature's own length then whether or not large animals have a more spacious existence will depend how the surface area of the earth increases as we use smaller and smaller units of measurement. If the surface area of the earth increases with a dimension $D = 2.25$, all creatures will have, roughly speaking, the same spatial experience, but if the average value of D falls below 2.25 then smaller animals will be more and more tightly packed. At any rate, it is pleasing that Damuth's value corresponds roughly to the hide and seek experiment since it offers a new way that architecture can imitate nature, that is it can have a similar fractal dimension to the natural world.

The effect of dimension on perceived distances

Lee (1970) established that perceived distances are a function of direction. In his experiment students were asked to estimate walking distances in Dundee to familiar landmarks both into and out of town. His results showed that distances were overestimated and that, other things being equal, distances towards the centre were seen as shorter than those out of town and that women thought they were closer than men. Lee thought that a number of factors were involved in this effect, but that the most important was that journeys to the centre seemed shorter because they were more rewarding, citing experimental evidence that if objects are viewed with favour they are seen as closer. The fact that women felt the centre to be relatively closer than men he attributed to women being more interested than men in shopping. He added however, that the complexity of a journey might lead to shorter time estimates, by analogy with the perception of filled and unfilled temporal intervals; that is, eventful journeys seem shorter.

If this latter explanation is accepted then it is reasonable to suppose that a walk in an area of high dimension will seem shorter than one of equal length in an area of low dimension. Furthermore a smaller person will perceive the distance as being less than a tall person. If this is correct then the reason that Lee found that women felt that the centre was nearer than men was not due to their liking shops more but because they are generally not as tall as men. It would be interesting to test this by repeating Lee's experiment and correlating the results, not with sex, but with size. If this is accepted, then it is evidence that regions with a high dimension are more attractive than those with a low dimension.

Character of spaces of different dimensions

What is the relationship between the number of places in a space and its dimension? Generally, a high value for the dimension and number of places will go together, although it is possible to imagine a space where they are independent. An example would be a theatre where the number of places is constant over a wide range of sizes of people provided there is social pressure to sit normally. Usually, however, if places are created by furniture and complications in the plan then a high density of places and a high dimension will go together. Bearing these points in mind we may make some observations about the character of spaces of different dimension.

A space of low dimension will be a poor place to play hide and seek. It will appear similar to children and adults and will seem crowded with only a few people in it. There will be few features, windows, for example, which people could use to locate themselves. What little furniture there is will be fixed to the floor. A window, if there is one, will offer no view and so will prevent someone from locating themselves in relation to something outside. Uniform artificial lighting will not allow places to be distinguished by their illumination. Alternatively, a space may be minimised by forcing people to sit in a particular way and not allowing them to move, as on an aeroplane. Acoustically the space will be reverberant so that people will have to stand apart if they do not wish to be overheard and a single person could fill the place with sound. In an extreme case, a space so cold and uncomfortable that it presents no places to live comfortably may have a dimension of zero, corresponding to a point.

These requirements are fulfilled by a smooth-walled prison cell with a single artificial light and a bed screwed to the floor, although a minimalist art gallery with a single work of art and one diffuse roof light will do nearly as well. At a cocktail party held in such a place, people would have to define the space they occupy by their relation to other partygoers; that is close to someone, far away from another. There being nowhere to sit out, such a gathering will exaggerate the importance of qualities such as looks, manners, and stamina. This is not, of course, to say that a minimalist style is necessarily bad, after all it may have qualities of purity and simplicity which are appropriate for certain uses, but in general a space which restricts the ways in which it may be occupied will risk being oppressive.

On the other hand, a space of high dimension is a good place to play hide and seek and will appear larger to a child than a grown-up. It will not be possible to see the whole space at a glance but it will reveal itself as one walks around. It will contain subspaces, alcoves, bays, and so forth over several orders of size and the space may be configured in many ways by moving furniture, screens, or curtains. There will be many routes across it. There will be elements at all scales over the whole space, from human sized to spaces for small ornaments. Lighting will come from several directions and spaces will change their character as they are lit differently during the day. The space will seem to soak people up, upon entering it will be hard to guess how many people are present. There will be many interesting objects, art works, and mirrors in relation to which a person may define a place to feel secure, not only in the sense of defensible space but in giving sufficient reason for standing in a particular place. An example of such a space is shown in figure 7, and although the trustees may have reasonable misgivings about allowing an experiment with a horde of children to measure its dimension, it is difficult to imagine that the Soane Museum in London can be beaten.

Figure 7. The Soane Museum, a space of high dimension.

Discussion—how to create space from nothing

If the argument presented here is accepted it means that the subjective size of a space may, to a certain extent, be independent of its area in square metres, and it will be possible, so to speak, to create usable space from nothing by increasing its dimension. This should not be surprising, as this is what many designers do. To take one example from many in his work, Alexander et al (1977) give advice for designing urban spaces which may be interpreted as advice to increase their fractal dimension. They observe that the density of occupation gives an indication of the liveliness of a space—150 to 300 square feet per person being a good figure—but since most public spaces do not reach this level they offer suggestions for making them more attractive and accommodating. Because people tend to go to the perimeter of spaces they advise that edges should be scalloped to make pockets where people will linger and that activity areas should project into the square wherever major paths do not go. They add that public spaces should have an off-centre middle, a tree, or a fountain along with a high spot with steps for sitting and a view. Their advice has the effect of creating places which may be identified and named, in effect making the square larger according to the definition of space used here. Yet they are at pains to restrict the width of the square so that it does not go beyond a width of about 75 feet, after which contact is lost between people. Alexander et al are, in effect, creating places without enlarging the space.

The number of dwellings per hectare which should be allowed in new developments is a contentious issue. It might be the case, however, that the arguments are based on an inappropriate unit of density since it does not take account of the plastic possibilities of space to accommodate more places if it is designed with a high fractal dimension. Housing at 100 dwellings per hectare in old Edinburgh may not, in fact, seem any more cramped than an estate of detached houses at 25 per hectare if the unit of measurement was, let us say, the number of places per unit area where a child could find to hide. Estates where houses are widely spaced to satisfy overlooking rules or the convenience of cars are, in effect, designed by arranging large objects in space and will likely have a low dimension which goes some way to accounting for the feeling of empty oppressiveness of some modern housing developments, a point of view which would support the opinions of the so-called 'new urbanists'. I intend to continue this line of research by counting places in urban environments rather than square metres to see whether our usual methods of assessing space give a fair representation of how the space is actually used.

References

Alexander C, Ishikawa S, Silverstein M, 1977 *A Pattern Language* (Oxford University Press, New York) pp 311 – 314 and 597 – 608
Batty M, Longley P, 1994, "Urban growth and form", in *Fractal Cities* (Academic Press, London) pp 228 – 273
Bolles-Wilson, 1993, "Stadtschloss competition" *Architecture Today* **42** (October) 13
Bovill C, 1996 *Fractal Geometry in Architecture and Design* (Birkhäuser, Boston, MA)
Damuth J, 1981, "Population density and body size in mammals" *Nature* **290** 23 April, pages 699 – 700
Gregory R L, 1998 *Eye and Brain* (Oxford University Press, Oxford)
Kiemle M, 1967 *Åsthetische Probleme der Architektur unter dem Aspekt der Informationsåsthetik* [Information content and aesthetics in architecture] (Schnelle, Quickborn, Germany)
Lee T, 1970, "Perceived distance as a function of direction in the city" *Environment and Behaviour* June, pages 40 – 51
McMahon T, Binner J, 1983 *On Size and Life* (W H Freeman, New York)
Mandelbrot B, 1983 *The Fractal Geometry of Nature* (W H Freeman, San Francisco, CA)
Peters R H, 1983 *The Ecological Implications of Body Size* (Cambridge University Press, Cambridge)
Salingaros N A, West B J, 1999, "A universal rule for the distribution of sizes" *Environment and Planning B: Planning and Design* **26** 909 – 923
von Buttlar A, Selig H, Wetzig A, 1972, "Erhaltenswerte Stadtbildelemente des Münchener Cityrandgebiets Lehel" [Architectural features worth preserving in the Munich suburb of Lehel] *Deutche Kunst und Denkmalplege* **30**(1) 65 – 71 (a description of this work may be found in: Wright L, 1973, "How many bits" *Architectural Review* **153** (April) 251 – 252)